Kuwait University   Math 102   March 25th, 1999

Math. & Comp. Sci. Dept.   First Examination   Time: 75 minutes.

Calculators and Mobile Phones are not allowed.

Each part counts 5 points.

1. Let $f(x) = \dfrac{1 - e^{\sqrt{x}}}{1 + e^{\sqrt{x}}}$. Show that f^{-1} exists and find $f^{-1}(x)$. Find the domain and range of f^{-1}.

2. Find the exact value of $\cos\left(\tan^{-1}\left(\dfrac{1}{4}\right) - \dfrac{\pi}{3}\right)$.

3. Prove that $\ln(a^r) = r \ln a$, where $a > 0$ and r is a rational number.

4. Find $\dfrac{dy}{dx}$ if

    a) $y = \left(\dfrac{x^2 \sin^{-1} x}{(1 - 2x)e^{2x}}\right)^3$.

    b) $\sinh(e^{xy}) + \ln(\cosh y) = x$.

5. Evaluate

    a) $\displaystyle\int x^2 4^{-x^3}\, dx$.

    b) $\displaystyle\int \dfrac{1}{\sqrt{e^{2x} - 4}}\, dx$.

    c) $\displaystyle\int \dfrac{x^3 + x}{x^4 + 1}\, dx$.

6. Find the limits

    a) $\displaystyle\lim_{x \to 0} \dfrac{\tan^{-1}(x^2)}{1 - \cos x}$.

    b) $\displaystyle\lim_{x \to \infty} e^x \ln(1 + e^{-x})$.